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Crack detection using nonlinear output frequency response functions

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Abstract

The new concept of nonlinear output frequency response functions (NOFRFs) is introduced in this paper to detect cracks in beams using frequency domain information. The results show that the NOFRFs are a sensitive indicator of the presence of cracks providing the excitation is of an appropriate intensity. They also constitute an indicative and promising basis, which must further be explored, so that the method can be used for the detection of cracks in beams with applications in structural fault diagnosis.

1. Introduction

Fatigue cracks are a potential source of catastrophic structural failure. To avoid failure caused by cracks, many researchers have performed extensive investigations to develop structural integrity monitoring techniques. Most of these techniques are based on vibration measurements and analysis because, in most cases, vibration-based methods can offer an effective and convenient way to detect fatigue cracks. Generally, vibration-based methods can be classified into two categories: linear and nonlinear approaches. Linear approaches detect the presence of cracks in a target object by monitoring changes in the resonant frequencies [1,2], in the mode shapes [3–5] or in the damping factors [6,7]. However, several researchers have shown [8–10] that, linear detection procedures are not always reliable and they typically show a low sensitivity to defects. For example, in Ref. [9], the numerical results show that the presence of a crack, which makes up about 10–20% of the undamaged cross-sectional area, reduces the natural frequencies of a beam by only 0.6–1.9%. Some factors, which may cause difficulties when using linear methods for crack detection in practice, have been discussed in Ref. [10]. Over recent years, increasing attention has been focused on the application of the nonlinear methods to detect the presence of cracks [9–14]. When a cracked object is subjected to a harmonic input, the appearance of superharmonic components and subharmonic resonances may be observed. An early investigation of using these nonlinear phenomena to detect the presence of cracks was done by Tsyfanskii et al. [11], they found that in the process of vibration of a bar with a closing crack, sub- and superharmonic

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resonance vibrations arise and they are quite distinct even in the case of a rather shallow crack. In Refs. [9–14], these phenomena are termed 'the nonlinear effects'. In Ref. [9], Bovsunovsky and Surace claimed that nonlinear effects are more sensitive to the presence of a crack than the change in natural frequencies, or mode shapes. These authors also studied the influence of damping on the nonlinear effects. Based on subharmonic resonances, Tsyfansky and Beresnevich [10] developed a new approach for the detection of fatigue cracks in flexible, geometrically nonlinear beam-type structural elements. Later, they [12] used the same procedure to detect cracks in aircraft wings. Sundermeyer and Weaver [13] studied the forced response of a bilinear model subjected to an excitation with two frequencies, and based on these results they further exploited the weakly nonlinear character of a cracked beam to determine the crack location. In Ref. [14], Saavedra and Cuitino studied dynamic behaviors of different multibeam systems containing a transverse crack theoretically and experimentally, and gave many results regarding which nonlinear effects would be useful for crack detection.

In summary, as indicated in previous studies by several authors, nonlinear analysis-based methods are often much more sensitive to the presence of cracks than linear vibration-based methods. The research reported in this paper is devoted to the introduction of the concept of the nonlinear output frequency response functions (NOFRFs) [15,16] and its application of this for crack detection. NOFRFs are a new concept developed recently by the authors, which allows the analysis of nonlinear systems to be implemented in a similar manner to linear system frequency response analysis. This provides great insight into how nonlinear phenomena such as the generation of new frequencies occur. This paper is focused on an experimental study to demonstrate that the NOFRFs are a good indicator of the presence of cracks in a beam, with the aim of establishing a basis for the use of NOFRFs in structural defect diagnosis in engineering practice.

The paper is organized as follows. Section 2 gives a brief introduction to the new concept of NOFRFs. The widely used breathing crack model is discussed in Section 3. The experimental study showing the application of the NOFRFs to crack detection is presented in Section 4. Finally, conclusions are given in Section 5.

2. Nonlinear output frequency response functions (NOFRFs)

2.1. NOFRFs under general inputs

NOFRFs were recently proposed and used to investigate the behavior of structures with polynomial-type nonlinearities [15]. The definition of NOFRFs is based on the Volterra series theory [17] of nonlinear systems. The Volterra series extends the well-known convolution integral description for linear systems to a series of multidimensional convolution integrals, which can be used to represent a wide class of nonlinear systems [16].

Consider the class of nonlinear systems which are stable at zero equilibrium and which can be described in the neighborhood of the equilibrium by the Volterra series

$$y(t) = \sum_{n=1}^{N} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_n(\tau_1, \ldots, \tau_n) \prod_{i=1}^{n} u(t - \tau_i)\, \mathrm{d}\tau_i, \tag{1}$$

where $y(t)$ and $u(t)$ are the output and input of the system, $h_n(\tau_1, \ldots, \tau_n)$ is the nth-order Volterra kernel, and N denotes the maximum order of the system nonlinearity. Lang and Billings [16] derived an expression for the output frequency response of this class of nonlinear systems to a general input. The result is

$$\begin{cases} Y(\mathrm{j}\omega) \sum_{n=1}^{N} Y_n(\mathrm{j}\omega), \quad \text{for } \forall\omega, \\ Y_n(\mathrm{j}\omega) = \dfrac{1/\sqrt{n}}{(2\pi)^{n-1}} \int_{\omega_1+\cdots+\omega_n=\omega} H_n(\mathrm{j}\omega_1, \ldots, \mathrm{j}\omega_n) \prod_{i=1}^{n} U(\mathrm{j}\omega_i)\, \mathrm{d}\sigma_{n\omega}. \end{cases} \tag{2}$$

This expression reveals how nonlinear mechanisms operate on the input spectra to produce the system output frequency response. In Eq. (2), $Y(\mathrm{j}\omega)$ is the spectrum of the system output, $Y_n(\mathrm{j}\omega)$ represents the nth-order output frequency response of the system

$$H_n(\mathrm{j}\omega_1, \ldots, \mathrm{j}\omega_n) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_n(\tau_1, \ldots, \tau_n) \mathrm{e}^{-(\omega_1\tau_1+\cdots+\omega_n\tau_n)\mathrm{j}}\, \mathrm{d}\tau_1 \cdots \mathrm{d}\tau_n \tag{3}$$

is the nth-order generalized frequency response function (GFRF) [16], and

$$\int_{\omega_1+\cdots+\omega_n=\omega} H_n(j\omega_1,\ldots,j\omega_n)\prod_{i=1}^{n} U(j\omega_i)\,d\sigma_{n\omega}$$

denotes the integration of $H_n(j\omega_1,\ldots,j\omega_n)\prod_{i=1}^{n} U(j\omega_i)$ over the n-dimensional hyper-plane $\omega_1+\cdots+\omega_n=\omega$. Eq. (2) is a natural extension of the well-known linear relationship $Y(j\omega)=H(j\omega)U(j\omega)$, where $H(j\omega)$ is the frequency response function (FRF), for the nonlinear case.

For linear systems, the possible output frequencies are the same as the frequencies in the input. For nonlinear systems described by Eq. (1), however, the relationship between the input and output frequencies is more complicated. Given the frequency range of an input, the output frequencies of system (1) can be determined using the explicit expression derived by Lang and Billings in Ref. [16].

Based on the above results for the output frequency response of nonlinear systems, a new concept known as the NOFRF was recently introduced by Lang and Billings [15]. The NOFRF is defined as

$$G_n(j\omega)=\frac{\int_{\omega_1+\cdots+\omega_n=\omega} H_n(j\omega_1,\ldots,j\omega_n)\prod_{i=1}^{n} U(j\omega_i)\,d\sigma_{n\omega}}{\int_{\omega_1+\cdots+\omega_n=\omega}\prod_{i=1}^{n} U(j\omega_i)\,d\sigma_{n\omega}} \tag{4}$$

under the condition that

$$U_n(j\omega)=\int_{\omega_1+\cdots+\omega_n=\omega}\prod_{i=1}^{n} U(j\omega_i)\,d\sigma_{n\omega}\neq 0. \tag{5}$$

Notice that $G_n(j\omega)$ is valid over the frequency range of $U_n(j\omega)$, which can be determined using the algorithm in Ref. [16].

By introducing the NOFRFs $G_n(j\omega)$, $n=1,\ldots,N$, Eq. (2) can be written as

$$Y(j\omega)=\sum_{n=1}^{N} Y_n(j\omega)=\sum_{n=1}^{N} G_n(j\omega)U_n(j\omega), \tag{6}$$

which is similar to the description of the output frequency response for linear systems. For a linear system, the relationship between $Y(j\omega)$ and $U(j\omega)$ can be illustrated as shown in Fig. 1. Similarly, the nonlinear system input and output relationship of Eq. (6) can be illustrated as shown in Fig. 2.

The NOFRFs reflect a combined contribution of the system properties and the input to the system output frequency response behavior. It can be seen from Eq. (4) that $G_n(j\omega)$ depends not only on H_n ($n=1,\ldots,N$) but also on the input $U(j\omega)$. For any structure, the dynamical properties are determined by the GFRFs H_n ($n=1,\ldots,N$). However, from Eq. (3) it can be seen that the GFRF is multidimensional [18,19], which can make the GFRFs difficult to measure, display and interpret in practice. Feijoo et al. [20,21] demonstrated that the Volterra series can be described by a series of associated linear equations whose corresponding associated FRFs are easier to analyze and interpret than the GFRFs. According to Eq. (4), the NOFRF $G_n(j\omega)$ is

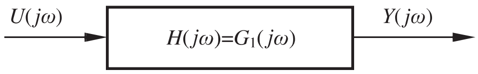

Fig. 1. The output frequency response of a linear system.

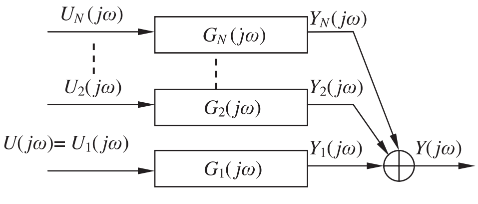

Fig. 2. The output frequency response of a nonlinear system.

a weighted sum of $H_n(\mathrm{j}\omega_1,\ldots,\mathrm{j}\omega_n)$ over $\omega_1+\cdots+\omega_n=\omega$ with the weights depending on the test input. Therefore, $G_n(\mathrm{j}\omega)$ can be used as an alternative representation of the dynamical properties described by H_n. The most important property of the NOFRF $G_n(\mathrm{j}\omega)$ is that it is one-dimensional, and thus allows the analysis of nonlinear systems to be implemented in a convenient manner similar to the analysis of linear systems. Moreover, there is an effective algorithm [15] available which allows the estimation of the NOFRFs to be implemented directly using system input–output data. This algorithm is briefly introduced below.

Rewrite Eq. (6) as

$$[Y(\mathrm{j}\omega)]=[U_1(\mathrm{j}\omega),\ldots,U_N(\mathrm{j}\omega)][G(\mathrm{j}\omega)], \tag{7}$$

where $[G(\mathrm{j}\omega)]=[G_1(\mathrm{j}\omega),\ldots,G_N(\mathrm{j}\omega)]^{\mathrm{T}}$.

Consider the case of $u(t)=\alpha u^*(t)$ where α is a constant and $u^*(t)$ is the input signal under which the NOFRFs of the system are to be evaluated, then

$$\begin{aligned}U_n(\mathrm{j}\omega)&=\frac{1/\sqrt{n}}{(2\pi)^{n-1}}\int_{\omega_1+\cdots+\omega_n=\omega}\prod_{i=1}^{n}U(\mathrm{j}\omega_i)\,\mathrm{d}\sigma_{n\omega}\\&=\alpha^n\frac{1/\sqrt{n}}{(2\pi)^{n-1}}\int_{\omega_1+\cdots+\omega_n=\omega}\prod_{i=1}^{n}U^*(\mathrm{j}\omega_i)\,\mathrm{d}\sigma_{n\omega}=\alpha^nU_n^*(\mathrm{j}\omega),\end{aligned} \tag{8}$$

where $U^*(\mathrm{j}\omega)$ is the Fourier transform of $u^*(t)$ and

$$U_n^*(\mathrm{j}\omega)=\frac{1/\sqrt{n}}{(2\pi)^{n-1}}\int_{\omega_1+\cdots+\omega_n=\omega}\prod_{i=1}^{n}U^*(\mathrm{j}\omega_i)\,\mathrm{d}\sigma_{n\omega}.$$

In this case, it is known from Eq. (7) that

$$[Y(\mathrm{j}\omega)]=\left[\alpha U_1^*(\mathrm{j}\omega),\ldots,\alpha^N U_N^*(\mathrm{j}\omega)\right][G^*(\mathrm{j}\omega)], \tag{9}$$

where $[G^*(\mathrm{j}\omega)]=[G_1^*(\mathrm{j}\omega),\ldots,G_N^*(\mathrm{j}\omega)]^{\mathrm{T}}$ which are the NOFRFs to be evaluated.

Excite the system under study $\bar{N}$ times using the input signals $\alpha_iu^*(t)$, $i=1,\ldots,\bar{N}$, where $\bar{N}\geqslant N$ and $\alpha_{\bar{N}},\alpha_{\bar{N}-1},\ldots,\alpha_1$ are constants which satisfy the condition

$$\alpha_{\bar{N}}>\alpha_{\bar{N}-1}>\cdots>\alpha_1>0,$$

so that $\bar{N}$ output frequency responses $Y^i(\mathrm{j}\omega)$, $i=1,\ldots,\bar{N}$ can be generated for the system under study. From Eq. (9), it is known that the output frequency responses can be related to the NOFRFs to be evaluated as below:

$$\mathbf{Y}^{1,\ldots,\bar{N}}(\mathrm{j}\omega)=\mathbf{AU}^{1,\ldots,\bar{N}}(\mathrm{j}\omega)[G^*(\mathrm{j}\omega)], \tag{10}$$

where

$$\mathbf{Y}^{1,\ldots,\bar{N}}(\mathrm{j}\omega)=\left[Y^1(\mathrm{j}\omega),\ldots,Y^{\bar{N}}(\mathrm{j}\omega)\right]^{\mathrm{T}} \tag{11}$$

and

$$\mathbf{AU}^{1,\ldots,\bar{N}}(\mathrm{j}\omega)=\begin{bmatrix}\alpha_1U_1^*(\mathrm{j}\omega),\ldots,\alpha_1^NU_N^*(\mathrm{j}\omega)\\ \vdots\\ \alpha_{\bar{N}}U_1^*(\mathrm{j}\omega),\ldots,\alpha_{\bar{N}}^NU_N^*(\mathrm{j}\omega)\end{bmatrix}.$$

Consequently, the values of the NOFRFs, $G_1^*(\mathrm{j}\omega), \ldots, G_N^*(\mathrm{j}\omega)$, can be determined using a least-squares-based approach as

$$\begin{aligned}[G^*(\mathrm{j}\omega)] &= \left[G_1^*(\mathrm{j}\omega), \ldots, G_N^*(\mathrm{j}\omega)\right]^{\mathrm{T}} \\ &= \left[\left(\mathbf{AU}^{1,\ldots,\bar{N}}(\mathrm{j}\omega)\right)^{\mathrm{T}}\left(\mathbf{AU}^{1,\ldots,\bar{N}}(\mathrm{j}\omega)\right)\right]^{-1}\left(\mathbf{AU}^{1,\ldots,\bar{N}}(\mathrm{j}\omega)\right)^{\mathrm{T}}\mathbf{Y}^{1,\ldots,\bar{N}}(\mathrm{j}\omega). \end{aligned} \tag{12}$$

In the present study, this algorithm was fulfilled using the Matlab 7.0.

2.2. NOFRFs under harmonic inputs

Harmonic inputs are pure sinusoidal signals which have been widely used for the dynamic testing of many engineering structures. Therefore, it is necessary to extend the NOFRF concept to the harmonic input case.

When system (1) is subject to a harmonic input

$$u(t) = A\cos(\omega_F t + \beta), \tag{13}$$

Lang and Billings [16] showed that Eq. (1) can be expressed as

$$Y(\mathrm{j}\omega) = \sum_{n=1}^{N} Y_n(\mathrm{j}\omega) = \sum_{n=1}^{N}\left(\frac{1}{2^n}\sum_{\omega_{k_1}+\cdots+\omega_{kn}=\omega} H_n(\mathrm{j}\omega_{k_1}, \ldots, \mathrm{j}\omega_{k_n})A(\mathrm{j}\omega_{k_1})\cdots A(\mathrm{j}\omega_{k_n})\right), \tag{14}$$

where

$$A(\mathrm{j}\omega) = \begin{cases} |A|\mathrm{e}^{j\,\mathrm{sign}(k)\beta}, & \text{if } \omega \in \{k\omega_F, k = \pm 1\}, \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

Defining the frequency components of nth-order output of the system as Ω_n, then according to Eq. (14), the frequency components in the system output can be expressed as

$$\Omega = \bigcup_{n=1}^{N} \Omega_n, \tag{16}$$

and Ω_n is determined by the set of frequencies

$$\left\{\omega = \omega_{k_1} + \cdots + \omega_{k_n} | \omega_{k_i} = \pm\omega_F, \quad i = 1, \ldots, n\right\}. \tag{17}$$

From Eq. (17), it is known that if all $\omega_{k_1}, \ldots, \omega_{k_n}$ are taken as $-\omega_F$, then $\omega = -n\omega_{\mathrm{F}}$. If k of these are taken as ω_F, then $\omega = (-n + 2k)\omega_F$. The maximal k is n. Therefore the possible frequency components of $Y_n(\mathrm{j}\omega)$ are

$$\Omega_n = \left\{(-n + 2k)\omega_F, \quad k = 0, 1, \ldots, n\right\}, \tag{18}$$

Moreover, it is easy to deduce that

$$\Omega = \bigcup_{n=1}^{N} \Omega_n = \{k\omega_F, k = -N, \ldots, -1, 0, 1, \ldots, N\}. \tag{19}$$

Eq. (19) explains why some superharmonic components will be generated when a nonlinear system is subjected to a harmonic excitation. It is worthy noting here that the harmonic components are symmetrical about the frequency axis; therefore, in the following, only those components with positive frequencies will be considered.

The NOFRFs defined in Eq. (4) can be extended to the case of harmonic inputs as

$$G_n^H(\mathrm{j}\omega) = \frac{(1/2^n)\sum_{\omega_{k_1}+\cdots+\omega_{kn}=\omega} H_n(\mathrm{j}\omega_{k_1}, \ldots, \mathrm{j}\omega_{k_n})A(\mathrm{j}\omega_{k_1})\cdots A(\mathrm{j}\omega_{k_n})}{(1/2^n)\sum_{\omega_{k_1}+\cdots+\omega_{kn}=\omega} A(\mathrm{j}\omega_{k_1})\cdots A(\mathrm{j}\omega_{k_n})}, \quad n = 1, \ldots, N, \tag{20}$$

under the condition that

$$A_n(\mathrm{j}\omega) = \frac{1}{2^n} \sum_{\omega_{k_1}+\cdots+\omega_{kn}=\omega} A(\mathrm{j}\omega_{k_1}) \cdots A(\mathrm{j}\omega_{k_n}) \neq 0. \tag{21}$$

Obviously, $G_n^H(\mathrm{j}\omega)$ is only valid over Ω_n defined by Eq. (18). Consequently, the output spectrum $Y(\mathrm{j}\omega)$ of nonlinear systems under a harmonic input can be expressed as

$$Y(\mathrm{j}\omega) = \sum_{n=1}^{N} Y_n(\mathrm{j}\omega) = \sum_{n=1}^{N} G_n^H(\mathrm{j}\omega) A_n(\mathrm{j}\omega). \tag{22}$$

When k of the n frequencies of $\omega_{k_1}, \ldots, \omega_{k_n}$ are taken as ω_F and the remainder are as $-\omega_F$, substituting Eq. (15) into Eq. (21) yields,

$$A_n(\mathrm{j}(-n+2k)\omega_F) = \frac{1}{2^n} C_n^k |A|^n \,\mathrm{e}^{\mathrm{j}(-n+2k)\beta}. \tag{23}$$

Thus $G_n^H(\mathrm{j}\omega)$ becomes

$$\begin{aligned} G_n^H(\mathrm{j}(-n+2k)\omega_F) &= \frac{(1/2^n) H_n\left(\overbrace{\mathrm{j}\omega_F, \ldots, \mathrm{j}\omega_F}^{k}, \overbrace{-\mathrm{j}\omega_F, \ldots, -\mathrm{j}\omega_F}^{n-k}\right) C_n^k |A|^n \,\mathrm{e}^{\mathrm{j}(-n+2k)\beta}}{(1/2^n) C_n^k |A|^n \,\mathrm{e}^{\mathrm{j}(-n+2k)\beta}}, \\ &= H_n\left(\overbrace{\mathrm{j}\omega_F, \ldots, \mathrm{j}\omega_F}^{k}, \overbrace{-\mathrm{j}\omega_F, \ldots, -\mathrm{j}\omega_F}^{n-k}\right), \end{aligned} \tag{24}$$

where $H_n(\mathrm{j}\omega_1, \ldots, \mathrm{j}\omega_n)$ is assumed to be a symmetric function. Therefore, in this case, $G_n^H(\mathrm{j}\omega)$ over the nth-order output frequency range $\Omega_n = \{(-n+2k)\omega_F,\ k = 0, 1, \ldots, n\}$ is equal to the GFRF $H_n(\mathrm{j}\omega_1, \ldots, \mathrm{j}\omega_n)$ evaluated at $\omega_1 = \cdots = \omega_k = \omega_F$, $\omega_{k+1} = \cdots = \omega_n = -\omega_F$, $k = 0, \ldots, n$.

This result indicates that the concept of NOFRFs can represent, to a certain extent, the dynamic characteristics of a nonlinear system under investigation, and may therefore be suitable for fault detection of mechanical or civil structures based on the difference of the structural dynamics in the fault and fault-free situations.

3. Nonlinearity of cracked beams

The presence of a crack in a beam will introduce a local flexibility that affects its dynamic response. During vibrations, a crack does not remain always open; it will open and close over time depending on the loading conditions and vibration amplitudes. If the static deflection due to loading on a cracked beam (e.g., body weight of the beam) is larger than the vibration amplitude, then the crack may remain in one condition all the time, always open or always closed depending on the position of the crack. In this case, the cracked beam may be described as a linear system. If the static deflection is small, then the crack may open and close over time depending on the vibration amplitude. In this case, the cracked beam will behave as a nonlinear system, and nonlinear effects will be present in the output response [22].

Using the finite-element method, the dynamic equation of a crack-free beam can be written as [22]

$$[M]\{\ddot{U}\} + [K]\{U\} = \{F\}, \tag{25}$$

where $[M]$ is the mass matrix, $[K]$ is the stiffness matrix, $\{U\}$ is the displacement vector and $\{F\}$ is the load vector. For a cracked beam, when the crack is open, an additional stiffness $-[\Delta K]$ is introduced, and Eq. (25) changes to

$$[M]\{\ddot{U}\} + [K - \Delta K]\{U\} = \{F\}. \tag{26}$$

In the study of cracks, a breathing crack is often considered, and it is assumed that when the bending moment changes sign, cracks change from open to closed, or from closed to open. Therefore, a cracked beam can behave like a bilinear nonlinear system, as described by [22]

$$\begin{cases} [M]\{\ddot{U}\} + [K]\{U\} = \{F\} & \text{if crack is closed,} \\ [M]\{\ddot{U}\} + [K - \Delta K]\{U\} = \{F\} & \text{if crack is open.} \end{cases} \quad (27)$$

The bilinear system (27) is a typical nonlinear system. Numerical studies have shown that this bilinear model can explain the nonlinear phenomena of the generation of superharmonic components, which have been observed in the output response of cracked beams subjected to a harmonic input. A crack-free beam behaves linearly as described by Eq. (25) and can thus be analyzed simply using the well-established FRF. However, the FRF cannot be effectively used to explain the nonlinear phenomena that are characteristic of a cracked beam. This is because the linear FRF-based approaches basically monitor the changes of structures in the resonant frequencies or in the mode shapes. However, the presence of cracks will often not induce a significant change in these structural characteristics. In order to solve this problem, the concept of NOFRFs was introduced in Ref. [15] to describe the behavior of cracked structures. The results showed that the NOFRFs can provide an explicit explanation for the generation of superharmonic components from a bilinear system subjected to a harmonic excitation. Based on the NOFRF description, the difference between cracked and crack-free beams can be illustrated as shown in Fig. 3. Fig. 3 indicates that the NOFRF concept can be used to more effectively distinguish the cracked and crack-free situations in structures.

4. An experimental study of crack detection using NOFRFs

A Volterra series-based method was used in Ref. [23] to analyze the vibration of a cracked beam where the higher-order transform function (HOTF) of a cracked cantilevered beam was estimated. The HOTF was defined as the ratio between the output spectrum $Y(\mathrm{j}\omega)$ and $U_n(\mathrm{j}\omega)$ for a particular n of interest and is based on the Volterra series of nonlinear systems. Compared to NOFRFs, the HOTF is not a theoretically well-established concept although under certain conditions, NOFRFs and HOTF can be related to each other. In this section, the application of the concept of NOFRFs for crack detection is investigated based on an experimental study.

The experimental test rig is shown in Fig. 4, which mainly consists of a shaker to generate the excitation, a clamp to fix the beam on the shaker, a beam and an accelerometer mounted at the free end of the beam to

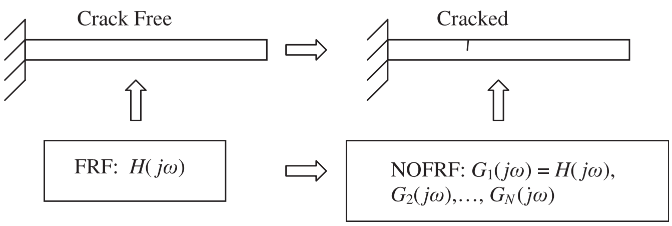

Fig. 3. The difference between crack-free and cracked beams.

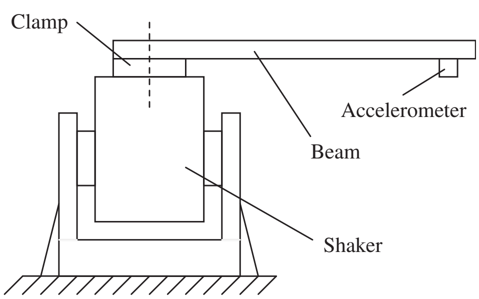

Fig. 4. Experimental test rig.

measure the acceleration. Three specimen beams were tested: one crack-free, one with a slight crack defect (the ratio α between the crack depth and the thickness of the beam was about 0.2), and one with a deep crack (the ratio α was about 0.4). The three specimen beams have same geometrical ($450 \times 20 \times 6\,\text{mm}^3$) and material properties (Young's modulus $E = 207 \times 10^9\,\text{Pa}$; mass density $\rho = 7850\,\text{kg/m}^3$; Poisson ratio $v = 0.33$). According to the requirements for estimating the NOFRFs up to fourth order, four inputs with the same waveform but different intensities will be needed to excite the system, respectively. For the harmonic input case, by substituting Eq. (23) into Eq. (22), it is known that the frequency components of the output can be written as

$$Y(\mathrm{j}\omega_F) = G_1^H(\mathrm{j}\omega_F)A_1(\mathrm{j}\omega_F) + G_3^H(\mathrm{j}\omega_F)A_3(\mathrm{j}\omega_F), \tag{28}$$

$$Y(\mathrm{j}2\omega_F) = G_2^H(\mathrm{j}2\omega_F)A_2(\mathrm{j}2\omega_F) + G_4^H(\mathrm{j}2\omega_F)A_4(\mathrm{j}2\omega_F), \tag{29}$$

$$Y(\mathrm{j}3\omega_F) = G_3^H(\mathrm{j}3\omega_F)A_3(\mathrm{j}3\omega_F), \tag{30}$$

$$Y(\mathrm{j}4\omega_F) = G_4^H(\mathrm{j}4\omega_F)A_4(\mathrm{j}4\omega_F). \tag{31}$$

From Eqs. (28)–(31), it can be seen that two different inputs with the same waveform but different intensities are sufficient to estimate the NOFRFs up to fourth order. Therefore, in this study, two different inputs were used in each test. Considering the fact that the extensity of the excitation forces may affect the nonlinearity of cracked beams, for example, a small excitation force may only make the cracks open partly while a strong excitation could make the cracks open fully, to make sure cracks are at the same status during one test, the intensities of the two inputs were chosen such that the intensities did not differ from each other considerably. The frequency ω_F of the harmonic excitation was 200 Hz, and the vibration signals were sampled using an accelerometer at the sample rate of 8 kHz.

Figs. 5–7 show the FFT spectra of three sets of output responses which were sampled from three specimen beams under different excitation intensities: small excitations, moderate excitations and strong excitations where the shaker forces are about 10, 30 and 50 N, respectively. In these figures, the first four harmonic

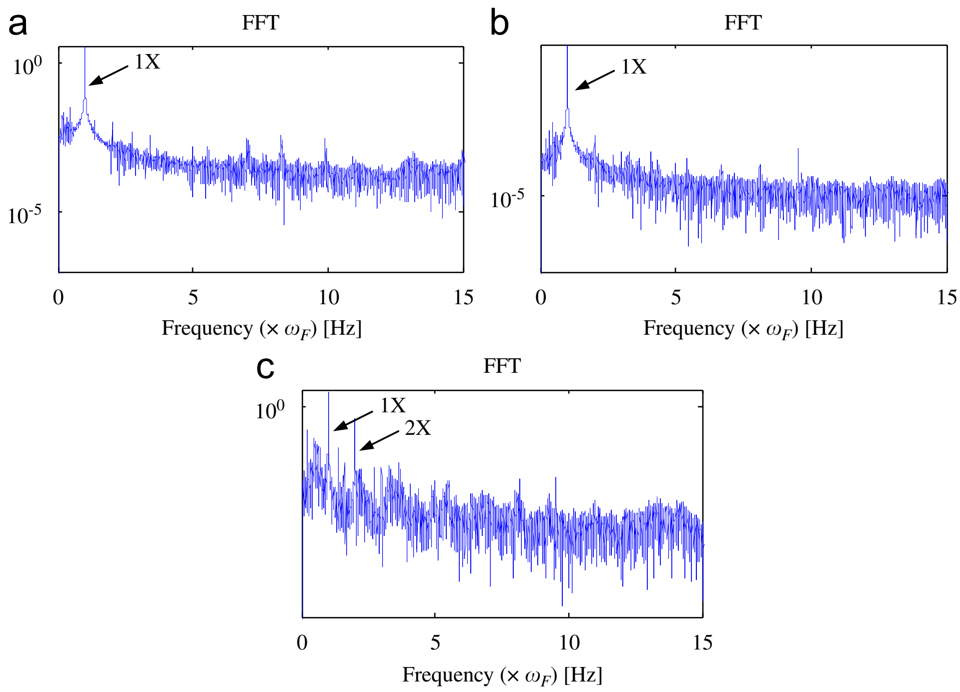

Fig. 5. The output responses under small excitations: (a) crack-free, (b) small crack and (c) large crack.

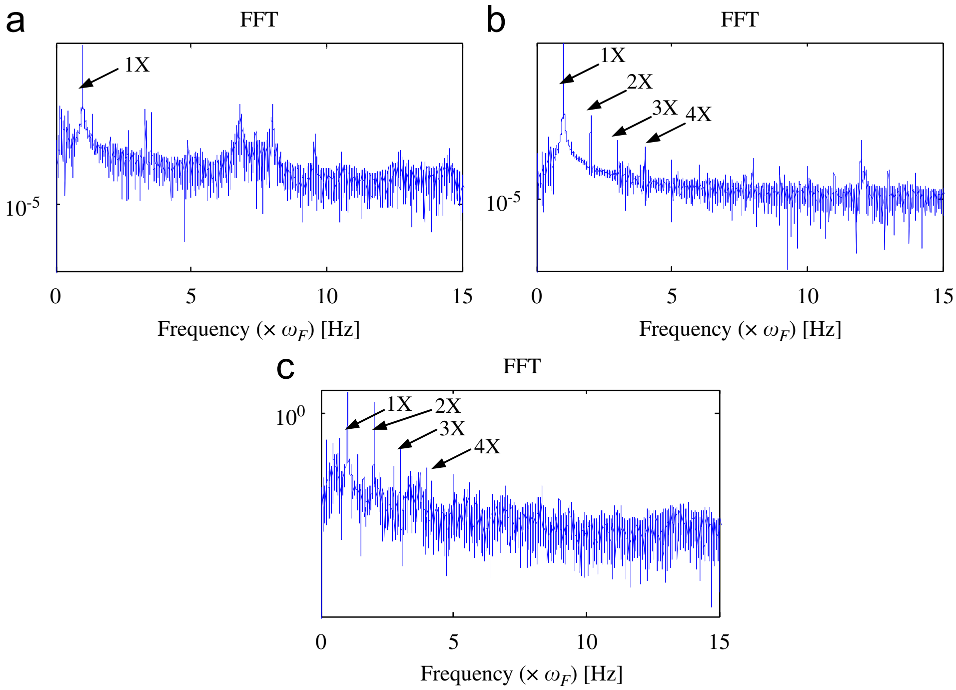

Fig. 6. The output responses under moderate excitations: (a) crack-free, (b) small crack and (c) large crack.

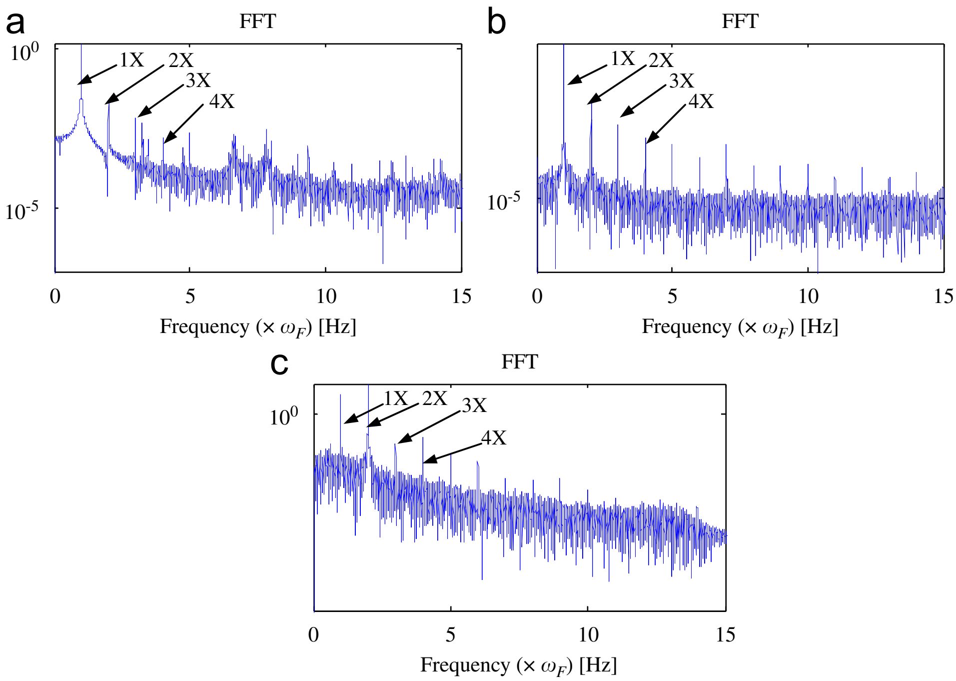

Fig. 7. The output responses under strong excitations: (a) crack-free, (b) small crack and (c) large crack.

components are marked by $1\times$, $2\times$, $3\times$ and $4\times$, respectively. Fig. 5 shows that under small excitations the nonlinear effects are very weak for both the crack-free beam and the beam with a small crack as the superharmonic components of the spectra are too small to be observed. However, Fig. 6 shows that the second harmonic component has a large amplitude when the beam has a large crack. The spectra in Fig. 6 show that, under moderate excitations, the nonlinear effect is still quite weak for the crack-free beam; however, the nonlinear effect becomes noticeable for both the two cracked beams as the superharmonic components up to fifth order are observable in the output spectra. Fig. 7 shows the output spectra of the vibration signals sampled under strong excitation. It can be seen that some superharmonic components and some irregular components appear in the output spectra of the crack-free beam. It is believed that in this case the strong excitations made the whole test rig behavior nonlinearly. For the two cracked beams, obviously, the nonlinear effect becomes significant, the superharmonic components are quite clear in the output spectra, especially in the output spectrum of the beam with a large crack where the second harmonic component is even larger than the fundamental harmonic component. These observation results indicate that the presence of cracks will induce the nonlinear effects in the output response, and the degree of nonlinearity depends on the intensity of the excitations.

Tables 1–3 give the NOFRF evaluation results for the three specimen beams. It is worth noting that all the NOFRFs in the tables have been normalized by dividing by $G_1^H(j\omega_F)$. It can be seen that, at the same excitation level, the NOFRFs of the beam with a large crack are always the largest, while for the beam with a small crack, and the crack-free beam the NOFRFs are always relatively small. This means that the behavior of the beam with a large crack is considerably more nonlinear than the crack-free beam. It also can be seen that, under small excitations, the NOFRFs of a beam with a small crack are quite small, even smaller than the NOFRFs of the crack-free beam under moderate and strong excitations. This is because small excitations may not cause the crack to open, and therefore the beam behaves just like a crack-free beam. In addition, the large values of the NOFRFs of the crack-free beam under strong excitations indicate that strong excitations made the test rig behave nonlinearly. Therefore, in a strong excitation case, the NOFRFs of the cracked beams reflect the combined nonlinear effects of the crack in the beam and the test rig, but compared to the nonlinear effect caused by a crack, the nonlinear effect from the test rig is less significant.

The results in Tables 1–3 show that the NOFRFs are a quite sensitive indicator of the presence of a crack as long as the excitation is strong enough to open the crack. Under small excitations, there are slight differences

Table 1
The estimated results under small excitations

NOFRFs	Crack free ($\alpha = 0$)	Small crack ($\alpha \approx 0.2$)	Large crack ($\alpha \approx 0.4$)
$\overline{G_1^H(j\omega_F)}$	1.00000000	1.00000000	1.00000000
$\overline{G_3^H(j\omega_F)}$	0.00100051	0.00263550	0.24103323
$\overline{G_2^H(j2\omega_F)}$	0.00060379	0.00110494	0.22953284
$\overline{G_4^H(j2\omega_F)}$	0.00007167	0.00184145	0.00755471
$\overline{G_3^H(j3\omega_F)}$	0.00002036	0.00060981	0.00369901
$\overline{G_4^H(j4\omega_F)}$	0.00001137	0.00004290	0.00078840

Table 2
The estimated results under moderate excitations

NOFRFs	Crack free ($\alpha = 0$)	Small crack ($\alpha \approx 0.2$)	Large crack ($\alpha \approx 0.4$)
$\overline{G_1^H(j\omega_F)}$	1.00000000	1.00000000	1.00000000
$\overline{G_3^H(j\omega_F)}$	0.00470760	0.02378153	0.28168441
$\overline{G_2^H(j2\omega_F)}$	0.00163466	0.02360642	0.24038379
$\overline{G_4^H(j2\omega_F)}$	0.00028966	0.00402358	0.04345069
$\overline{G_3^H(j3\omega_F)}$	0.00070613	0.00097110	0.02502336
$\overline{G_4^H(j4\omega_F)}$	0.00010685	0.00034400	0.00859901

Table 3
The estimated results under strong excitations

NOFRFs	Crack free ($\alpha = 0$)	Small crack ($\alpha \approx 0.2$)	Large crack ($\alpha \approx 0.4$)
$\overline{G_1^H(j\omega_F)}$	1.00000000	1.00000000	1.00000000
$\overline{G_3^H(j\omega_F)}$	0.00554740	0.02742614	0.05101400
$\overline{G_2^H(j2\omega_F)}$	0.00991495	0.03443461	0.49263892
$\overline{G_4^H(j2\omega_F)}$	0.00062943	0.00435547	0.21806627
$\overline{G_3^H(j3\omega_F)}$	0.00097094	0.00286848	0.06451821
$\overline{G_4^H(j4\omega_F)}$	0.00018662	0.00024341	0.03216557

between the NOFRFs of the crack-free beam and the slightly cracked beam, but the NOFRFs of the beam with a deep crack are much larger than the NOFRFs of the crack-free beam. Under moderate and large excitations, most of the NOFRFs of the cracked beams are much larger than the NOFRFs of the crack-free beam. Therefore, the NOFRFs are a good indicator of the presence of a crack. Moreover, it can be seen that the NOFRFs of the beam with a large crack are always larger than the NOFRFs of the beam with a small crack under the same excitation, which implies that the values of the NOFRFs can be regarded as an indicator of the crack size, larger NOFRFs inferring a larger crack size. The advantage of using the NOFRF results in Tables 1–3 is that single values are given for the NOFRFs which are much easier to compare and interpret compared to other frequency-based methods.

In summary, the experimental study shows that the NOFRFs are a sensitive indicator of the presence of cracks. To conduct the crack detection procedure using the NOFRFs, appropriate excitations should be employed. Ideally, the excitations should be strong enough to open a crack but should not be too strong; otherwise, the excitations will make the test rig behave nonlinearly and the difference between the NOFRFs evaluated in the cracked and crack-free situations may not be considerably different.

5. Conclusions and remarks

The new concept of NOFRFs has been introduced for fault detection. The importance of using the NOFRFs instead of the FRF to describe cracked beams in the frequency domain has been analyzed. Finally, an experimental study using the NOFRFs to detect cracks has been conducted for three specimens of beams, one without a crack, one with a small crack and one with a large crack. The results indicate that the NOFRFs are a quite sensitive indicator of the presence of cracks in a beam provided the excitations of appropriate intensities are employed, and the values of the computed NOFRFs are an indication of the crack size. Larger values of NOFRFs normally indicate larger crack sizes. The present study constitutes an indicative and promising basis for the detection of cracks in beams with applications in structural fault diagnosis.

The implementation of the method requires the excitation of the structure several times with different intensities. This can readily be achieved in practice. It should be mentioned that, in the present study, only crack ratios smaller than 0.4 were considered. More experiments on beams and other structures made out of different materials and with different crack depths are planned to be conducted in order to further validate the effectiveness of this method. A comparison with other established methods is also going to be studied. The results will be reported in future publications.

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